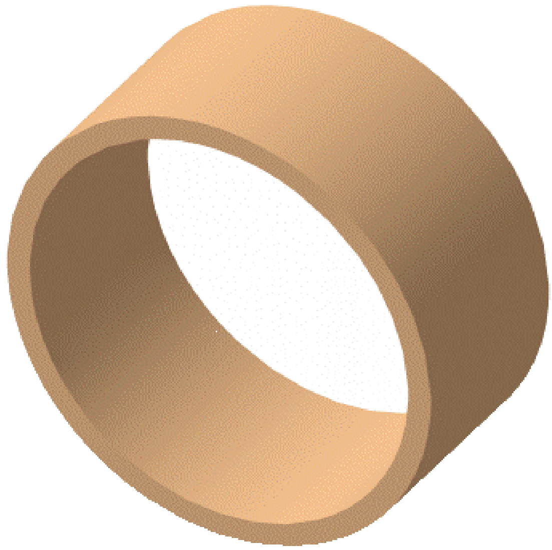
Rotating Cylindrical Shell



1) Purpose:

To calculate the stress and deflection of a lathe-turned maple shell held to a face plate at one end. The dynamics problem is first treated to find the internal pressure from centrifugal force. This pressure is then used in a static Finite Element Analysis. As you can see on the FEA pages, the deflection of this shell under normal lathe conditions is not measurable.

2) Shell Geometry:

OD	=	14.00 in	(7.00 in radius)
ID	=	12.50 in	(6.25 in radius)
Mid Diameter	=	13.25 in	(6.625 in radius)
Wall	=	0.75 in	
Depth	=	6.50 in	
Volume	=	202.93 in^3	
Density	=	.026 lb / in^3	(Maple = 3,100 lb / 70 ft^3)
Weight	=	5.28 lb	(202.93 x .026)
Mass	=	Weight / Acceleration of Gravity	
	=	5.28 lb / 32.16 ft / s^2	
	=	.164 Slugs	(lb · s^2 / ft)
	=	.014 lb · s^2 / in	
Rotation	=	650 RPM	(my slowest lathe speed)

3) Use techniques of Section 5.3 of "Analytical Mechanics," Grant R. Fowles, 1986, to determine the centrifugal force acting on a particle at the cylinder mid-radius.

Angular velocity at mid-radius:

$$\omega = \frac{650Rot}{\min} \times \frac{1\min}{60s} \times \frac{360°}{Rot} \times \frac{1rad}{57.30°} = 68.06radians/s$$

Centrifugal force at mid-radius:

$$F_{cent} = m\omega^2 r_{mid}$$

$$F_{cent} = \frac{.014lb \cdot s^2}{in} \times \left(\frac{68.06radians}{s}\right)^2 \times 6.625in = 429.63lb$$

4) Just as the mass of the shell is distributed, this force must also be distributed evenly across the area of the cylinder at mid-radius. This distributed centrifugal force is equivalent to a pressure applied to that area, so the problem can be solved with statics.

Surface area at mid-radius:

$$A_{mid} = 2\pi(r_{mid})Depth = 2\pi \times (6.625in) \times 6.50 = 270.57in^2$$

Centrifugal pressure on cylindrical surface at mid-radius:

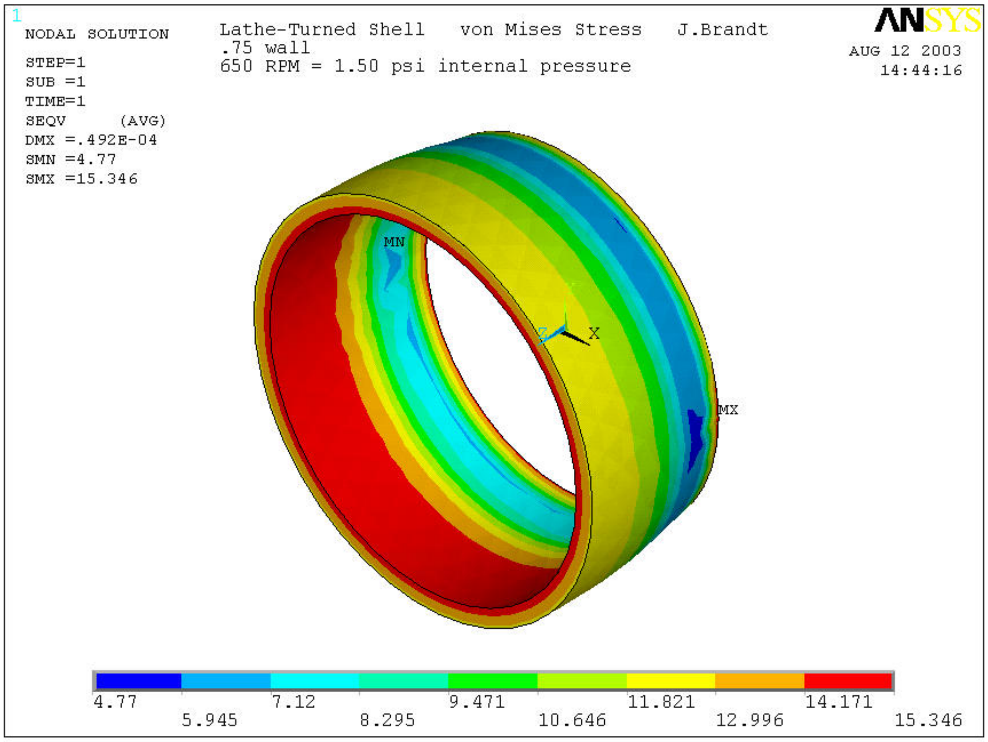
$$P_{mid} = \frac{F_{cent}}{A_{mid}} = \frac{429.63lb}{270.57in^2} = 1.59psi$$

Corresponding centrifugal pressure on cylindrical surface at inside radius:

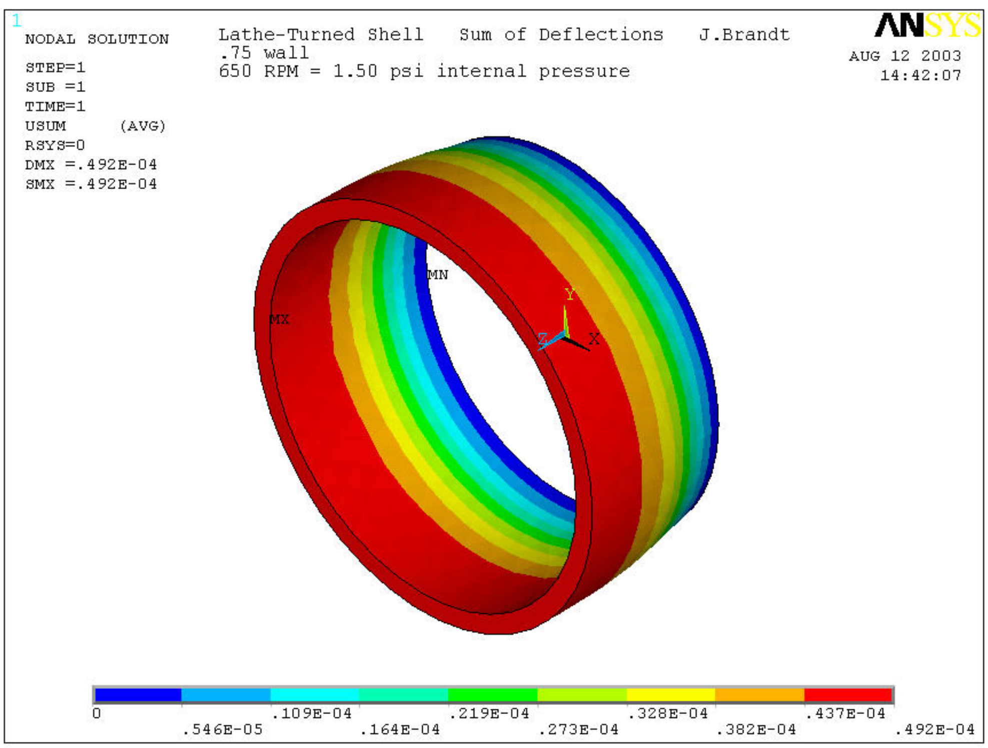
$$P_{ins} = \frac{(P_{mid})(A_{ins})}{A_{mid}} = \frac{(1.59psi)(255.25in^2)}{270.57in^2} = 1.50psi$$

5) Solve Ansys static Finite Element Analysis using geometry as shown on page 1 and properties below from "Mechanics of Materials," Russell C. Hibbeler, 1994. Assume one end of cylinder fixed to lathe face plate.

Internal Pressure	=	1.50 psi	(650 RPM)
Modulus of Elasticity	=	1,900,000 psi	(from Hibbeler)
Poisson's Ratio	=	0.29	(from Hibbeler)
Ultimate Strength	=	300 psi	(from Hibbeler)
Allowable Strength	=	75 psi	(4 to 1 safety factor)



Ansys FEA **Stress** – 0.75 wall – one end fixed @ 650 RPM = 15.346 psi



Ansys FEA **Deflection** – 0.75 wall – one end fixed @ 650 RPM = 0.000492 in

6) Conclusion:

Turning a fairly massive maple shell – 14.00" OD x 0.75" wall x 6.50" deep – on a lathe at a reasonable speed – 650 RPM – produces stress in the shell and results in an expansion of the diameter. Since the shell is assumed to be attached to a face plate, it is fixed at one end and this expansion is seen only at the free end of the shell.

Dynamics are used to find the centrifugal force on the rotating shell. This force is distributed around the shell, just as the shell's mass is distributed. This distributed force is considered as a pressure in a statics problem, using Ansys FEA to solve for stress and deflection (expansion at the free end).

In the shell turning at 650 RPM, the maximum stress is found to be 15.3 psi, well within the limit of allowable stress for this wood. The maximum deflection (expansion) at the free end of the shell is found to be 0.00049". This deflection is far below the expected machining tolerance of a lathe turned shell, and is virtually un-measurable.

7) Analysis Part 2:

In one of the forum threads, a turning speed of 10,000 RPM was mentioned. Since the problem was already set up, it was fairly easy to revise the internal pressure to solve for the stress and deflection of a shell at this angular velocity.

Here are the problem parameters:

Geometry	=	Same as on Page 1
Rotation	=	10,000 RPM

Angular velocity at mid-radius:

$$\omega = \frac{10{,}000 Rot}{\min} \times \frac{1\min}{60s} \times \frac{360°}{Rot} \times \frac{1rad}{57.30°} = 1{,}047 radians / s$$

Centrifugal force at mid-radius:

$$F_{cent} = m\omega^2 r_{mid}$$

$$F_{cent} = \frac{.014 lb \cdot s^2}{in} \times \left(\frac{1{,}047 radians}{s}\right)^2 \times 6.625 in = 101{,}673 lb$$

Surface area at mid-radius:

$$A_{mid} = 2\pi(r_{mid})Depth = 2\pi \times (6.625in) \times 6.50 = 270.57in^2$$

Centrifugal pressure on cylindrical surface at mid-radius:

$$P_{mid} = \frac{F_{cent}}{A_{mid}} = \frac{101{,}673lb}{270.57in^2} = 375.8psi$$

Corresponding centrifugal pressure on cylindrical surface at inside radius:

$$P_{ins} = \frac{(P_{mid})(A_{ins})}{A_{mid}} = \frac{(375.8psi)(255.25in^2)}{270.57in^2} = 354.5psi$$

Ansys static Finite Element Analysis parameters:

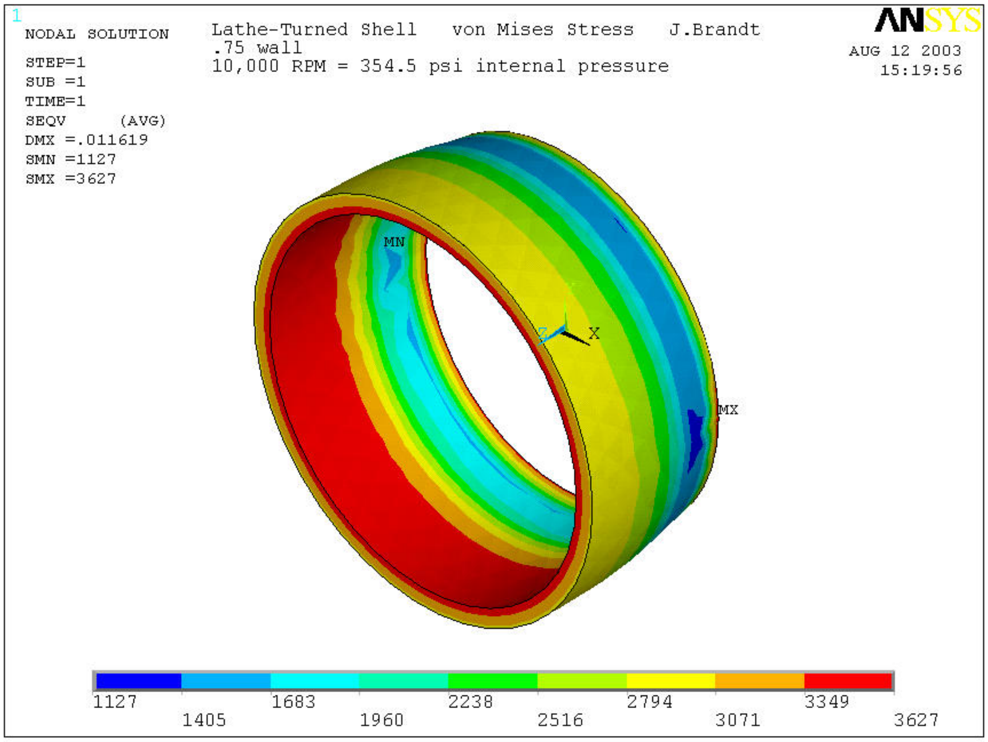
Internal Pressure	=	354.5 psi	(10,000 RPM)
Modulus of Elasticity	=	1,900,000 psi	(from Hibbeler)
Poisson's Ratio	=	0.29	(from Hibbeler)
Ultimate Strength	=	300 psi	(from Hibbeler)
Allowable Strength	=	75 psi	(4 to 1 safety factor)

8) Conclusion Part 2 (don't try this at home):

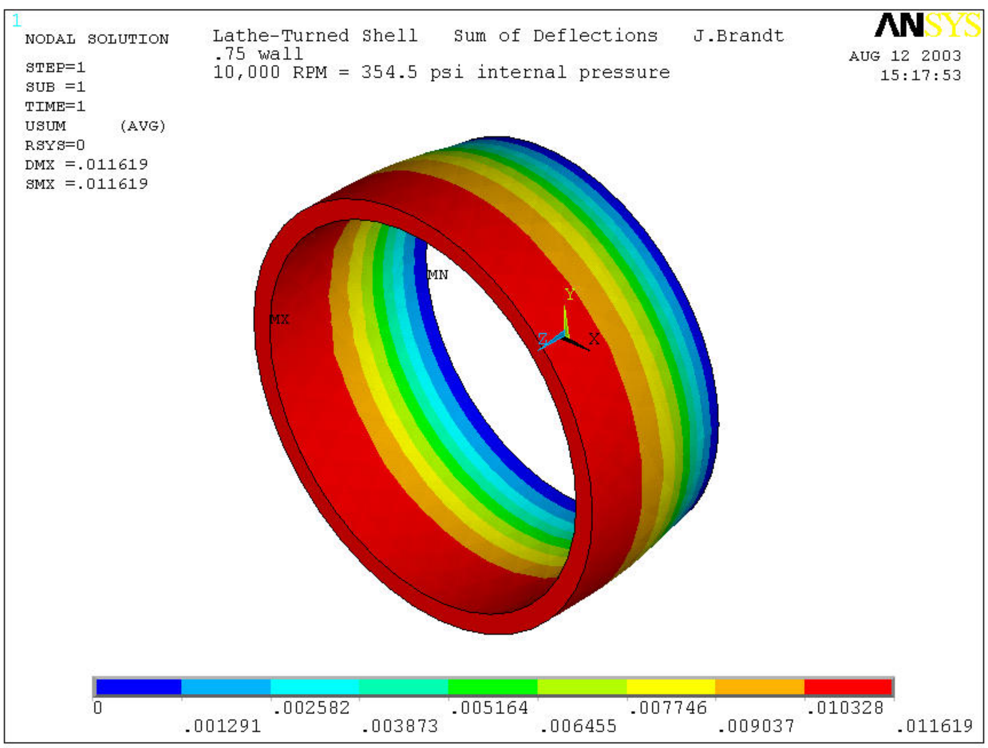
Fortunately, not many lathes will go to this rotational speed. In a shell turning at 10,000 RPM, the maximum stress is found to be 3,627 psi, 12 times the ultimate strength of this wood! Some type of banding or screwing method would be required to keep the shell from exploding off the lathe – *Repeat* – don't try this at home!

It is interesting to note that if the shell could be kept from self-destructing, even at 10,000 RPM the maximum deflection (expansion) at the free end of the shell is found to be 0.0116", less than 1/64". This deflection is larger than the expected tolerance of a lathe turned shell, but well within the range of error most of us will accept in a shell.

The 3/16" "centrifugal force taper" mentioned in the "Rhythm King drums and shells" thread is nothing more than poor quality control.



Ansys FEA **Stress** – 0.75 wall – one end fixed @ 10,000 RPM = 3,627 psi



Ansys FEA **Deflection** – 0.75 wall – one end fixed @ 10,000 RPM = 0.0116 in

9) Analysis Part 3: Consider a thinner wall shell

Here are the problem parameters:

OD	=	14.00 in	(7.00 in radius)
ID	=	13.50 in	(6.75 in radius)
Mid Diameter	=	13.75 in	(6.875 in radius)
Wall	=	0.25 in	
Depth	=	6.50 in	
Volume	=	70.19 in^3	
Density	=	.026 lb / in^3	(Maple = 3,100 lb / 70 ft^3)
Weight	=	1.82 lb	(70.19 x .026)
Mass	=	Weight / Acceleration of Gravity	
	=	1.82 lb / 32.16 ft / s^2	
	=	.057 Slugs	(lb · s^2 / ft)
	=	.0048 lb · s^2 / in	
Rotation	=	650 RPM	(my slowest lathe speed)

Angular velocity at mid-radius:

$$\omega = \frac{650 Rot}{\min} \times \frac{1 \min}{60 s} \times \frac{360^\circ}{Rot} \times \frac{1 rad}{57.30^\circ} = 68.06 radians / s$$

Centrifugal force at mid-radius:

$$F_{cent} = m\omega^2 r_{mid}$$

$$F_{cent} = \frac{.0048 lb \cdot s^2}{in} \times \left(\frac{68.06 radians}{s} \right)^2 \times 6.875 in = 152.86 lb$$

Surface area at mid-radius:

$$A_{mid} = 2\pi(r_{mid}) Depth = 2\pi \times (6.875 in) \times 6.50 = 280.78 in^2$$

Centrifugal pressure on cylindrical surface at mid-radius:

$$P_{mid} = \frac{F_{cent}}{A_{mid}} = \frac{152.86 lb}{280.78 in^2} = 0.54 psi$$

Corresponding centrifugal pressure on cylindrical surface at inside radius:

$$P_{ins} = \frac{(P_{mid})(A_{ins})}{A_{mid}} = \frac{(.54\,psi)(275.67\,in^2)}{280.78\,in^2} = 0.53\,psi$$

Ansys static Finite Element Analysis parameters:

Internal Pressure	=	0.53 psi	(650 RPM)
Modulus of Elasticity	=	1,900,000 psi	(from Hibbeler)
Poisson's Ratio	=	0.29	(from Hibbeler)
Ultimate Strength	=	300 psi	(from Hibbeler)
Allowable Strength	=	75 psi	(4 to 1 safety factor)

10) Conclusion Part 3:

For a thinner wall shell – 14.00" OD x 0.25" wall x 6.50" deep - turning at 650 RPM, the maximum stress is found to be 20.1 psi, 31% higher than the comparable heavy wall shell, but still well within the limit of allowable stress for this wood. The maximum deflection is found to be 0.00055", 12% higher but again virtually un-measurable.

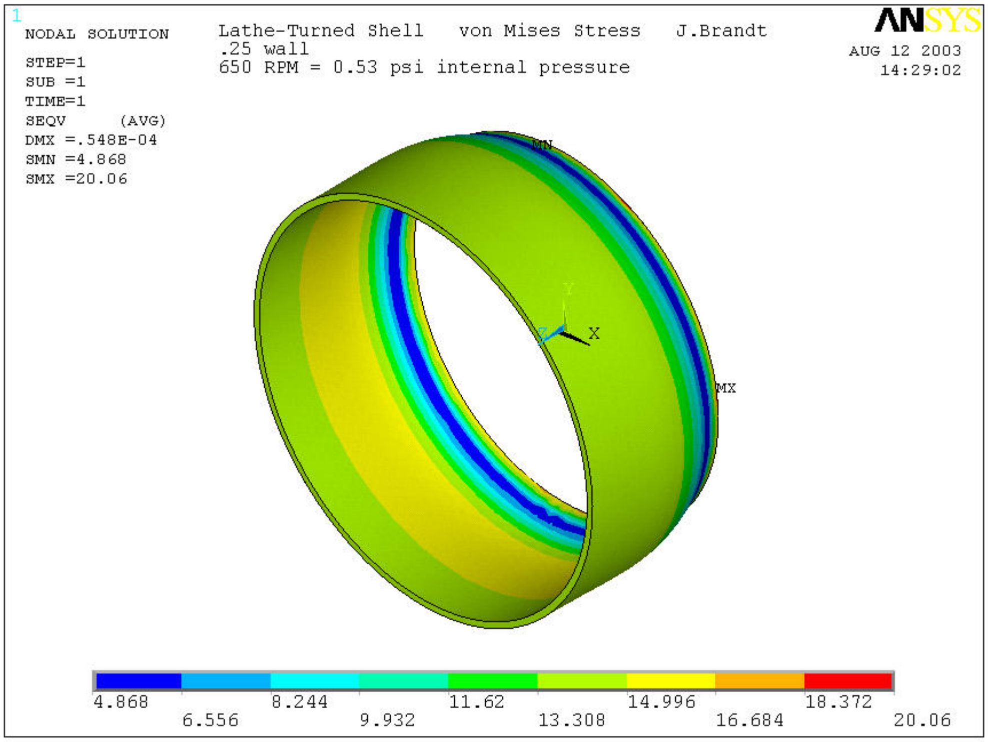
William brings up a couple of other points that are well worth further discussion. The availability of properties for wood is marginal at best. My model assumes an Isotropic material – one in which the properties are the same in all directions. This is certainly not true for wood, and it would be very difficult to predict or model the differences.

However, I contend that in spite of these shortcomings, the results shown are good to within an order of magnitude, and that even in the worst case, a lathe-turned shell should be perfect within the limits of measurable tolerances.

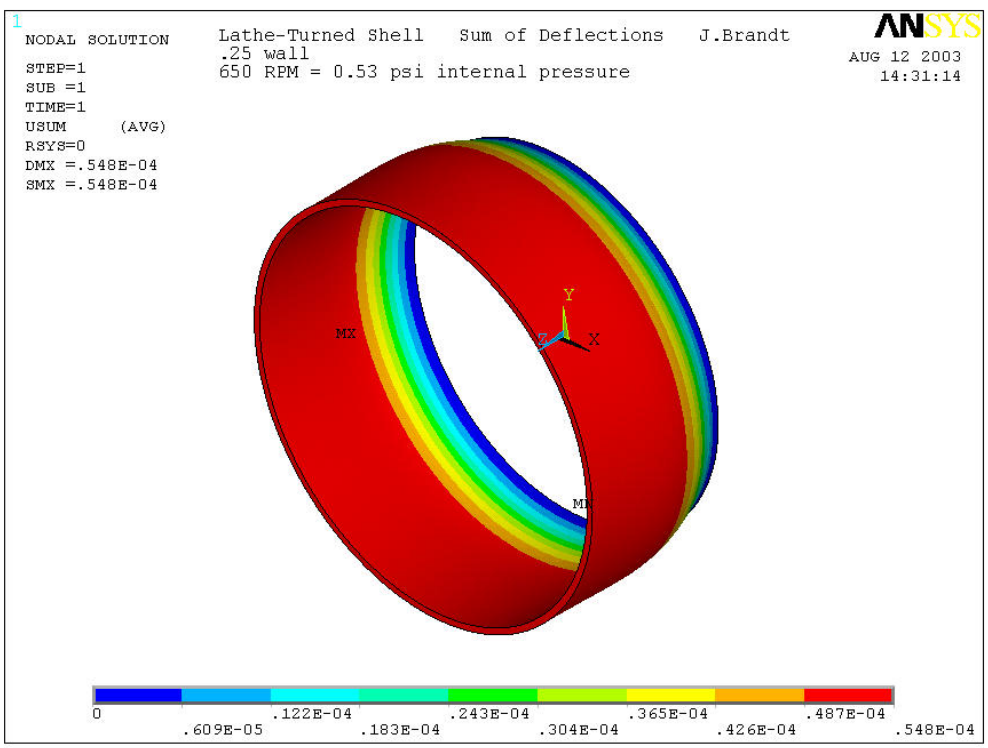
Finally, the lack of concentricity in a shell can produce a destructive wobble or resonant frequency while turning, especially when a shell is first mounted. All care should be taken to ensure that the raw shell is as close to concentric as possible before the machine is switched on. An adequate mounting method must be employed, and the operator should stand away from the work at first power on.

This condition of a non-concentric shell is too difficult for me to model with my knowledge of the analysis tools.

In the real world – SAFETY FIRST!



Ansys FEA **Stress** – 0.25 wall – one end fixed @ 650 RPM = 20.06 psi



Ansys FEA **Deflection** – 0.25 wall – one end fixed @ 650 RPM = 0.000548 in